# Cumulative elasticity factor derivation

Demand and supply curves are plotted on a graph with axes representing the market price for a commodity and the quantity demanded or supplied. They cross at the equilibrium point, whose coordinates represent the quantity that will be sold and the price at which each unit will sell, given an efficient market. The demand curve shifts to the left when there is a drop in demand unrelated to price. The system finds a new equilibrium where the new demand curve intersects the supply curve.
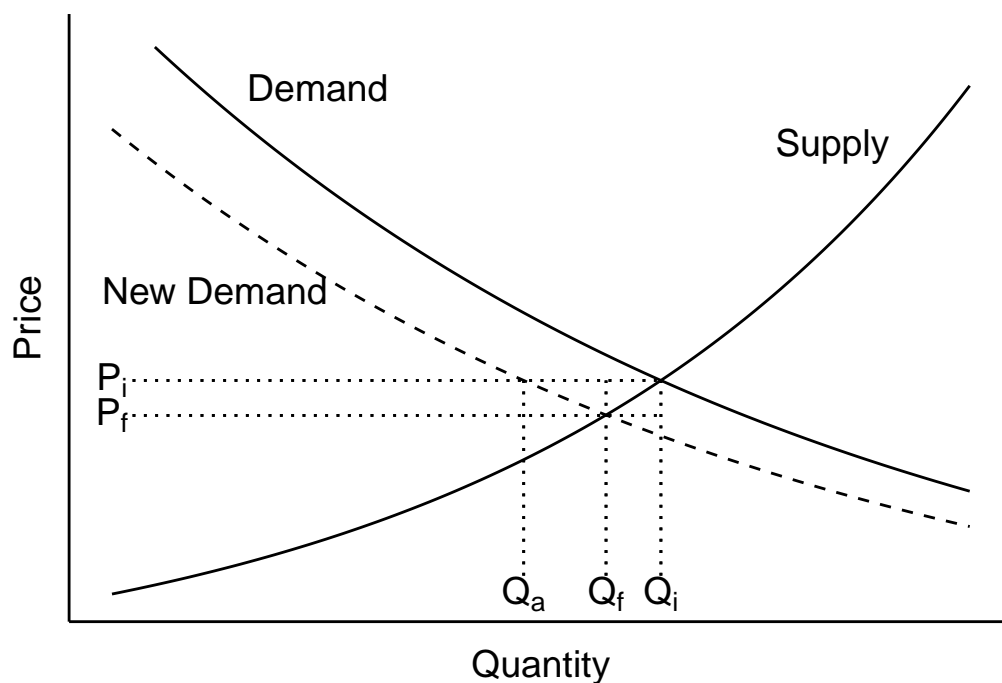
We want to determine Q_f in terms of Q_i and Q_a, because $(Q_i - Q_a)$ is the original change in the quantity demanded and $(Q_i - Q_f)$ is the final change in the equilibrium quantity supplied (or equivalently demanded).

To do that, we consider the triangular part of the plot formed by the new supply and demand curves and the horizontal line representing the original equilibrium price P_i. As we assume the shift is small, we can approximate the supply and demand curves in this region by straight lines corresponding to their slopes at the final equilibrium.
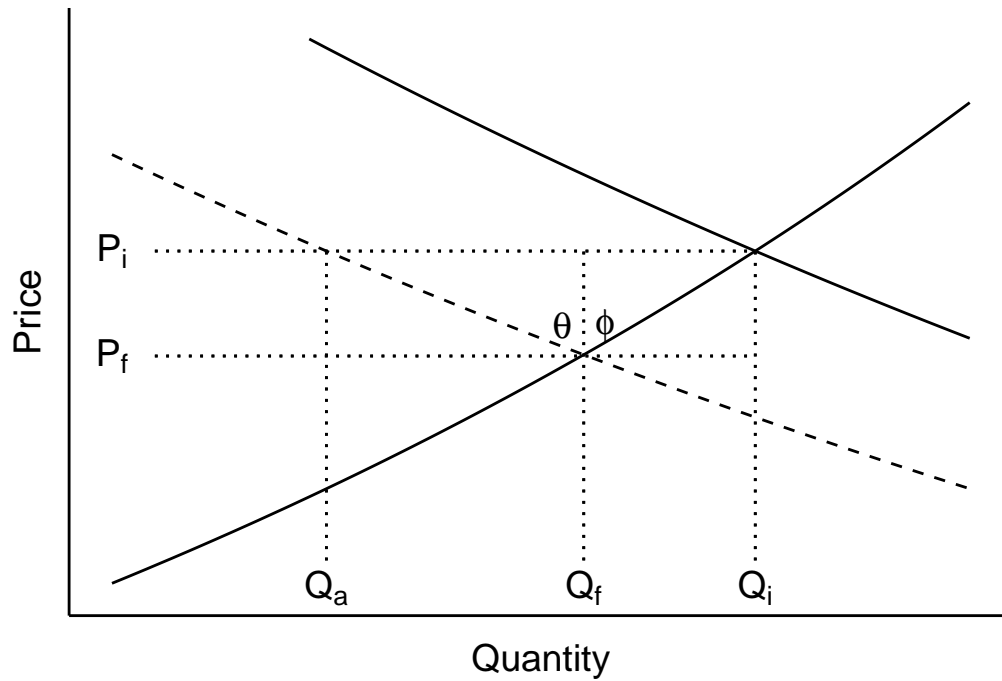
The definition of tangent gives us that

$$\tan\phi = \frac{Q_i - Q_f}{P_i - P_f}$$

This is approximately the inverse of the slope of the tangent line to the supply curve at the final equilibrium, which is equal to the supply elasticity at that point multiplied by $\frac{Q_f}{P_f}$ to provide the proper units.

$$\left.\frac{dQ^S}{dP}\right|_{(Q_f, P_f)} = \frac{Q_f}{P_f} \times (\text{supply elasticity})$$

Similarly,

$$\tan\theta = \frac{Q_f - Q_a}{P_i - P_f} \approx -\left.\frac{dQ^D}{dP}\right|_{(Q_f, P_f)} = -\frac{Q_f}{P_f} \times (\text{demand elasticity})$$

Combining these two approximations,

$$\frac{Q_i - Q_f}{Q_i - Q_a} = \frac{(P_i - P_f)\tan\phi}{(P_i - P_f)(\tan\phi + \tan\theta)} \approx \frac{\text{supply elasticity}}{\text{supply elasticity} - \text{demand elasticity}}$$

In the limit as $(Q_i - Q_a)$ approaches zero, the approximation is exact. This ratio allows us to calculate the difference between the quantity demanded at the old equilibrium and the quantity demanded at the new equilibrium, if we know how much change in quantity demanded occurred originally. It is called the *cumulative elasticity factor*.

Then

$$\text{final change in demand} = (\text{original change in demand}) \times (\text{cumulative elasticity factor}).$$